



Siberian Federal University
Laboratory of Complex Analysis and
Differential Equations

COMPLEX ANALYSIS AND DIFFERENTIAL EQUATIONS
INTERNATIONAL CONFERENCE
PROGRAM & ABSTRACTS

October 5 – 9, 2015
Saint Petersburg, Russia

THE CONFERENCE IS ORGANIZED BY

- Siberian Federal University (SibFU), Krasnoyarsk, Russia
- Laboratory of Complex Analysis and Differential Equations (SibFU), Krasnoyarsk, Russia

in accordance with State Contract №14.Y26.31.0006 (signed 19.03.2014) for a grant to support research at Siberian Federal University under supervision of a leading scientist.

Contacts:

E-mail: alexey.shchuplev@gmail.com

Site: <http://conf.sfu-kras.ru/en/math2015spb>

Phone: +7 391 246 98 86, +7 391 206 21 48

Address: Russia, 660041, Krasnoyarsk, pr. Svobodnyi, 79,
Siberian Federal University, Institute of Mathematics and Computer Science.

The timetable

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 – 9:30					
9:30 – 10:00	Registration and opening	Alain Yger		Grigory Mikhalkin	
10:00 – 10:30		Alexander Mkrtchyan		Dmitry Pohekutov	Alexander Mednykh
10:30 – 11:00	Maria Esteban	Coffee		Coffee	Coffee
11:00 – 11:30					Ilya Mednykh
11:30 – 12:00	Tatyana Suslina	Alexander Alexandrov		Alexander Loboda	Victor Chueshev
12:00 – 12:30					Vladimir Kulikov
12:30 – 13:00	Evgeny Korotyaev	Alain Hénaut		Evgeny Mikhalkin	
13:00 – 14:30	Lunch	Lunch		Lunch	Lunch
14:30 – 15:00					
15:00 – 15:30	Nikolay Filonov	Nikolay Shirokov		Timur Sadykov	
15:30 – 16:00					
16:00 – 16:30	Alexey Il'yin	Alekos Vidras		Yury Eliyashev	
16:30 – 17:00	Coffee	Evgeniya Yurieva		Ilya Lopatin	
17:00 – 17:30	Alexander Znamensky				

Schedule

Day I: Monday, October 5, 2015

- 9:30 – 10:00 *Registration and opening*
- 10:00 – 11:00 MARIA ESTEBAN : *Spectral estimates for Schrödinger operators on manifolds*
- 11:00 – 12:00 TATYANA SUSLINA : *Homogenization of nonstationary Schrödinger type equations with periodic coefficients*
- 12:00 – 13:00 EVGENY KOROTYAEV : *Schrödinger operators on the multidimensional lattice*
- LUNCH
- 14:30 – 15:30 NIKOLAY FILONOV : *Structure of edges of spectral bands of 2D periodic Schrödinger operator*
- 15:30 – 16:30 ALEXEY IL'YIN: *On a class of interpolation inequalities for differential and discrete operators and applications*
- COFFEE
- 17:00 – 17:30 ALEXANDER ZNAMENSKY : *A new proof of the Cauchy-Kovalevskaya theorem*

Day II: Tuesday, October 6, 2015

- 9:00 – 10:00 ALAIN YGER : *Multivariate residue calculus and arithmetics*
- 10:00 – 10:30 ALEXANDER MKRTCHYAN : *On continuability of multiple power series across the boundary of the convergence domain*
- COFFEE
- 11:00 – 12:00 ALEXANDER ALEXANDROV : *The Poincaré index and residue*
- 12:00 – 13:00 ALAIN HÉNAUT : *Infinitesimal symmetries for planar webs through meromorphic connections*
- LUNCH
- 14:30 – 15:30 NIKOLAY SHIROKOV : *Some weighted estimates for the $\bar{\partial}$ -equation*
- 15:30 – 16:30 ALEKOS VIDRAS : *Bergman-Weil expansions for holomorphic functions*
- 16:30 – 17:00 EVGENIYA YURIEVA : *Extension of analytic sets across the edge of the wedge in a degenerate position*

Day III: Wednesday, October 7, 2015

- 18:00 – *Conference dinner*

Day IV: Thursday, October 8, 2015

- 9:00 – 10:00 GRIGORY MIKHALKIN : *Logarithmic area of real plane curves and refined enumerative geometry*
- 10:00 – 10:30 DMITRY POCHEKUTOV : *Algebraicity of diagonals of rational power series*
- COFFEE
- 11:00 – 12:00 ALEXANDER LOBODA : *Coefficient approach to the classification problem for homogeneous hypersurfaces in \mathbb{C}^3*
- 12:00 – 13:00 EVGENY MIKHALKIN : *Parametrization of singular points of a general algebraic surface*
- LUNCH
- 14:30 – 15:30 TIMUR SADYKOV : *Extremal properties of hypergeometric polynomials*
- 15:30 – 16:30 YURY ELIYASHEV : *Generalized amoebas*
- 16:30 – 17:00 ILYA LOPATIN : *Variety of hypersurfaces with a given type of amoeba*

Day V: Friday, October 9, 2015

- 9:30 – 10:30 ALEXANDER MEDNYKH : *Graphs and Riemann surfaces*
- COFFEE
- 11:00 – 11:30 ILYA MEDNYKH : *On Jacobian of circulant graphs*
- 11:30 – 12:00 VICTOR CHUESHEV : *Variational formulas of the monodromy group for a third-order equation on a compact Riemann surface*
- 12:00 – 12:30 VLADIMIR KULIKOV : *Conditions for convergence of the Mellin-Barnes integral for solution to a system of algebraic equations*
- LUNCH

Titles and Abstracts

Day I: Monday, October 5, 2015

Maria Esteban *CNRS & Université Paris-Dauphine, Paris, France*

Spectral estimates for Schrödinger operators on manifolds

Abstract: In this talk I will present recent results about the estimates of the principal eigenvalue of Schrödinger operators on compact and non compact manifolds. These estimates are optimal and related to the proof of rigidity results for nonlinear elliptic PDEs on those manifolds. These results have been obtained in a series of papers in collaboration with J. Dolbeault, A. Laptev and M. Loss.

Tatyana Suslina *Saint Petersburg State University, Saint Petersburg, Russia*

Homogenization of nonstationary Schrödinger type equations with periodic coefficients

Abstract: In $L_2(\mathbb{R}^d; \mathbb{C}^n)$, we consider a selfadjoint strongly elliptic operator A_ε , $\varepsilon > 0$, given by the differential expression $b(\mathbf{D})^*g(\mathbf{x}/\varepsilon)b(\mathbf{D})$. Here $g(\mathbf{x})$ is a periodic bounded and positive definite matrix-valued function, and $b(\mathbf{D})$ is a first order differential operator. We study the behavior of the operator exponential e^{-itA_ε} for small ε . We prove that, as $\varepsilon \rightarrow 0$, the operator e^{-itA_ε} converges to e^{-itA^0} in the $(H^s(\mathbb{R}^d) \rightarrow L_2(\mathbb{R}^d))$ -operator norm (for a suitable s). Here $A^0 = b(\mathbf{D})^*g^0b(\mathbf{D})$ is the effective operator. Sharp order error estimates are obtained. The results are applied to study the Schrödinger type equation $i\partial_t u_\varepsilon(x, t) = (A_\varepsilon u_\varepsilon)(x, t)$.

Evgeny Korotyaev *Saint Petersburg State University, Saint Petersburg, Russia*

Schrödinger operators on the multidimensional lattice

Nikolay Filonov *PDMI RAS, Saint Petersburg, Russia*

Structure of edges of spectral bands of 2D periodic Schrödinger operator

Alexey Il'yin *Keldysh Institute of Applied Mathematics RAS, Moscow, Russia*

On a class of interpolation inequalities for differential and discrete operators and applications

Alexander Znamensky *Siberian Federal University, Krasnoyarsk, Russia*

A new proof of the Cauchy-Kovalevskaya theorem

Day II: Tuesday, October 6, 2015

Alain Yger *Université de Bordeaux, Bordeaux, France*

Multivariate residue calculus and arithmetics

Abstract: Given an algebraic subvariety $X \subset \mathbb{A}_{\mathbb{Q}}^n$ of pure dimension r , its degree d_X is an indicator of its geometric complexity while its canonical height $\hat{h}_X = \hat{h}(X)$ with respect to the standard inclusion $\mathbb{A}_{\mathbb{Q}} \hookrightarrow \mathbb{P}_{\mathbb{Q}}^n$ measures its arithmetic complexity. Given r elements (f_0, f_1, \dots, f_r) in $\mathbb{Z}[x_1, \dots, x_n]$ such that f_1, \dots, f_r define a 0-dimensional complete intersection on $X_{\mathbb{C}}$ and f_0, \dots, f_r do not share any common zero on $X_{\mathbb{C}}$, I will describe (in terms of d_X and \hat{h}_X) which estimates arithmetic intersection theory provides for the arithmetic complexity of (jointly) the residue morphisms

$$\sum_{|I|=r} g_I dx_I \quad (g_I \in \mathbb{Z}[x_1, \dots, x_n]) \longmapsto \text{Res} \left[\left(\frac{\sum_{|I|=r} g_I dx_I}{f_1^{\alpha+1}, \dots, f_r^{\alpha+1}} \right) / f_0 \right]_{X_{\mathbb{C}}} \quad (\alpha \in \mathbb{N}^r).$$

Such estimates should indeed govern the arithmetic complexity of Briançon-Skoda type theorems. These results have been obtained with Martín Sombra (University of Barcelona). I will also discuss the non-absolute complete intersection case and explain how some tools developed in our work in progress with Mats Andersson, Dennis Eriksson, Håkan Samuelsson Kalm and Elizabeth Wulcan could be transposed to the arithmetic setting. I will conclude this talk (if time permits) with natural questions in relation with arithmetic complexity that the “currential” point of view arises either in the projective or toric settings.

Alexander Mkrtychyan *Siberian Federal University, Krasnoyarsk, Russia*

On continuability of multiple power series across the boundary of the convergence domain

Alexander Aleksandrov *Institute of Control Sciences RAS, Moscow, Russia*

The Poincaré index and residue

Abstract: In 1886, H.Poincaré has introduced two basic concepts: the index of vector fields and the residue of differential forms on manifolds. We discuss some interesting relations between the corresponding notions in the context of the theory of complex spaces with arbitrary singularities. In particular, making use of a homological approach originated by X.Gómez-Mont, we show how to compute the local topological index of vector fields and holomorphic differential 1-forms given on complex space by elementary calculations. In the study of complete intersections our method is based on the construction of the Lebelt and the Cousin resolutions as well as on the simplest properties of the generalized and usual Koszul complexes, the regular meromorphic differential forms and the residue map. The general case is analyzed with the use of some additional considerations from deformation theory.

Alain Hénaut *CNRS & Université de Bordeaux, Bordeaux, France*

Infinitesimal symmetries for planar webs through meromorphic connections

Abstract: Planar web geometry deals with families of foliations locally in \mathbb{C}^2 or globally on $\mathbb{P}^2 := \mathbb{P}^2(\mathbb{C})$. A d -planar web $\mathcal{W}(d)$ is *implicitly* presented by the generic family of integral curves of respectively an analytic or an algebraic differential equation of the first order

$F(x, y, y') = 0$ with degree d . We study *infinitesimal symmetries* for such a $\mathcal{W}(d)$, that is the Lie algebra \mathfrak{g} of vector fields with local flow preserving all the leaves of $\mathcal{W}(d)$.

For $d \geq 3$, it is proved that \mathfrak{g} is a local system with rank 0, 1 or 3, outside the singular locus $|\Delta|$ defined by the p -discriminant of F . An explicit description of \mathfrak{g} is provided by using only the coefficients of F through connection methods. New invariants of the class of the differential equations defined by $F(x, y, y') = 0$ which extend the Blaschke curvature of a planar 3-web are introduced. Relations with W-curves in \mathbb{P}^2 of Klein-Lie and results of Élie Cartan are specified. Links with the *abelian relations* of $\mathcal{W}(d)$, that is special relations between the normals of its leaves inspired by Abel's addition theorem are given. Singularities and *residues* associated with \mathfrak{g} are also discussed.

The undertaken research can be also viewed as a part of the so-called qualitative study of differential equations. Methods and tools used illustrate the fertile interplay between complex analysis and algebraic geometry.

Nikolay Shirokov *Saint Petersburg State University & Higher School of Economics, Saint Petersburg, Russia*

Some weighted estimates for the $\bar{\partial}$ -equation

Alekos Vidras *University of Cyprus, Nicosia, Cyprus*

Bergman-Weil expansions for holomorphic functions

Abstract: Let $U \subset \mathbb{C}$ be a bounded domain with piecewise smooth boundary. Motivated by ideas coming from Several Complex Variables (in particular from the multiplicative theory of residue currents in the non complete intersection case as well as from weighted Cauchy or Cauchy-Pompeiu formulae), we state division interpolation formulas of the Lagrange type with respect to the powers of an ideal (f_1, \dots, f_m) (when $m > 1$) and derive from them a “balanced” Bergman-Weil type convergent expansion of h in the domain $\{z \in U : \|f(z)\| < \min_{\partial U} \|f(\zeta)\|\}$ in terms of $f^{\underline{k}}$, $\underline{k} \in \mathbb{N}^m$. Our approach is based on complex duality ideas and multidimensional residue theory (adapted here to the one variable setting).

Evgeniya Yurieva *Siberian Federal University, Krasnoyarsk, Russia*

Extension of analytic sets across the edge of the wedge in a degenerate position

Day IV: Thursday, October 8, 2015

Grigory Mikhalkin *Université de Genève, Geneva, Switzerland*

Logarithmic area of real plane curves and refined enumerative geometry

Abstract: We note that under certain conditions, the logarithmic images of real plane curves bound regions whose areas are half-integer multiples of π^2 . The half-integer number can be interpreted as a quantum state of the real curve and used to refine real enumerative geometry in a way consistent with the Block-Göttsche invariants from the tropical world.

Dmitry Pochekutov *Siberian Federal University, Krasnoyarsk, Russia*

Algebraicity of diagonals of rational power series

Alexander Loboda *Voronezh State University, Voronezh, Russia*

Coefficients approach to the classification problem for homogeneous hypersurfaces in \mathbb{C}^3

Abstract: The matter of a talk is the problem of classification of holomorphically homogeneous real hypersurfaces in 3-dimensional complex spaces. The method of describing of such surfaces in terms of the Taylor coefficients of their canonical (normal in sense of Moser) equations had been proposed early and allowed to classify the homogeneous surfaces with rich symmetry groups.

In general case of homogeneous surfaces with discrete isotropy groups the problem under discussion can be reduced to a system of polynomial equations with “small enough” number of variables.

Known examples of affine homogeneous hypersurfaces in 3-dimensional complex space are used to estimate the nearness of obtained necessary conditions of holomorphic homogeneity to the final solution of the problem.

Evgeny Mikhalkin *Siberian Federal University, Krasnoyarsk, Russia*

Parametrization of singular points of a general algebraic surface

Timur Sadykov *Plekhanov Russian University of Economics, Moscow, Russia*

Extremal properties of hypergeometric polynomials

Abstract: With any integer convex polytope $P \subset \mathbb{R}^n$ we associate a multivariate hypergeometric polynomial whose set of exponents is $\mathbb{Z}^n \cap P$. This polynomial is defined uniquely up to a constant multiple and satisfies a holonomic system of partial differential equations of Horn’s type. Special instances include numerous families of orthogonal polynomials in one and several variables. In the talk, we will discuss several extremal properties of multivariate polynomials defined in this way. In particular, we prove that the zero locus of any such polynomial is optimal in the sense of Forsberg-Passare-Tsikh.

Yury Eliyashev *Siberian Federal University, Krasnoyarsk & Moscow, Russia*

Generalized amoebas

Abstract: Recently I.Krichever defined a generalized amoeba of a complex curve. In my talk I will give the definition of generalized amoeba in higher dimensions and will describe some properties of this object.

Ilya Lopatin *Siberian Federal University, Krasnoyarsk, Russia*
Variety of hypersurfaces with a given type of amoeba

Day V: Friday, October 9, 2015

Alexander Mednykh *Sobolev Institute of Mathematics & Siberian Federal University, Novosibirsk, Russia*

Graphs and Riemann surfaces

Abstract: We give a short survey of old and new results about automorphism groups and branched coverings of graphs. The latter notion was introduced independently by T. D. Parsons, T. Pisanski, P. Jackson (1980), H. Urakawa (2000), B. Baker, S. Norine (2009) and others. The branched covering of graphs are also known as harmonic maps or vertically holomorphic maps of graphs. The main idea of the present talk is to create a parallel between classical results on branched covering of Riemann surfaces and those for graphs. We introduce the notion of harmonic action on a graph and discuss the Hurwitz type theorems for the groups acting harmonically. These results can be regarded as discrete analogues of the well known theorems by Hurwitz and Accola–Maclachlan. They, respectively, give sharp upper and lower bounds for the order of an automorphism group acting on a Riemann surface.

We present discrete versions of theorems by Wiman (1895), Oikawa (1956) and Arakawa (2000), which sharpen the Hurwitz upper bound for various classes of groups acting on a Riemann surface of given genus.

Then we define a hyperelliptic graph as two fold branched covering of a tree and a γ -hyperelliptic graph as two fold branched covering of a graph of genus γ . A few discrete versions of the well-known results on γ -hyperelliptic Riemann surface will be given.

Ilya Mednykh *Sobolev Institute of Mathematics & Siberian Federal University, Novosibirsk, Russia*

On Jacobian of circulant graphs

Abstract: We consider Jacobians of graphs as discrete analogues of Jacobians of Riemann surfaces. More precisely, Jacobian of graph is an Abelian group generated by flows satisfying the first and the second Kirchhoff rules. We define a circulant graph as the Cayley graph of a cyclic group. The family of circulant graphs is quite wide. It includes complete graphs, cyclic graphs, antiprism graphs, even prism graphs and Moebius ladder graph. We propose a new method to find the structure of Jacobians for a large subfamily of circulant graphs.

Victor Chueshev *Kemerovo State University, Kemerovo, Russia*

Variational formulas of the monodromy group for a third-order equation on a compact Riemann surface

Vladimir Kulikov *Siberian Federal University, Krasnoyarsk, Russia*

Conditions for convergence of the Mellin-Barnes integral for solution to a system of algebraic equations

Abstract: We consider the Mellin-Barnes integral that corresponds to the monomial function of a solution to a system of n algebraic equations in n unknowns. We obtain a necessary condition for the convergence domain of the integral to be non empty. For $n = 2, 3$ we prove that this condition is also sufficient.